Dispersion of Spinning Missiles due to Lift Nonaveraging

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A new source of cross-range dispersion is identified for spinning missiles during untrimmed flight. The dispersion is caused from wind-fixed moment disturbances that produce coupled perturbations in angle of attack and lift vector precession rate, in contrast to body-fixed moment-induced dispersion of a rolling, trimmed missile in which lift and roll variations generally are uncoupled. Small wind-oriented moments can affect the precession rate adversely and cause lift nonaveraging. The dispersion, in some cases, depends strongly on the initial motion conditions. Since roll rate has little influence on the lift vector precession rate, roll control is ineffective for limiting this type of dispersion, and effective control must provide some means of maintaining a steady, nonzero precession rate.

Nomenclature

$C_{L\theta}$ = aerodynamic lift force derivative
i = $\sqrt{-1}$
I = pitch or yaw moment of inertia
I_x = roll moment of inertia
J = aerodynamic jump (trajectory deflection angle)
L = aerodynamic lift force
L_θ = lift force derivative, $C_{L\theta}\, qS$
m = missile mass
m_t = M_t/I
m_w = $(-M_y + iM_p)/I$
M_p = in-plane disturbance moment (pitch)
M_r = roll moment
M_t = body-fixed trim moment
M_y = out-of-plane disturbance moment (yaw)
p = roll rate
p_{cr} = critical roll rate, $\omega/(1-\mu)^{1/2}$
q = dynamic pressure
r = cross-range dispersion
s = target range; Laplace transform variable
S = aerodynamic reference area
t = time
t_p = pulse duration
u = missile velocity
v, w = cross-range velocity components
V = complex cross-range velocity, $v + iw$
ΔV = net transverse velocity increment
Y, Z = cross-range coordinates
α = angle of attack
β = angle of sideslip
$\delta(t)$ = unit impulse function
ζ = aerodynamic pitch damping ratio
θ = angle of attack (Euler angle)
θ_0 = initial quasisteady angle of attack
θ_p = in-plane trim angle of attack
λ = precession rate ratio, $\dot{\psi}/\omega$
μ = inertia ratio, I_x/I
ξ = complex angle of attack, $\beta + i\alpha$
τ = precession period
ϕ = roll angle relative to wind (Euler angle); moment orientation angle
Φ = roll angle in inertial space
ψ = precession angle (Euler angle)
$\dot{\psi}$ = precession rate
$\dot{\psi}_p$ = quasisteady precession rate due to in-plane moment
$\dot{\psi}_{+,-}$ = precession modes
ω = natural pitch frequency

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Introduction

MOST statically stable missiles are given a nominal roll rate to average out lifting effects of configurational asymmetries. Even with steady roll, lift variations can cause dispersion from nonaveraging of the lift vector with each revolution of the missile. Conversely, steady lift with roll-rate variations or combinations of both lift and roll-rate variations can result in lift nonaveraging and dispersion. This class of impact error has been called "aerodynamic jump" and "roll-trim dispersion," and analyses of such phenomena have been limited to body-fixed configuration asymmetries for which the lift vector due to trim angle of attack rotates in inertial space at the missile roll rate.[1-11] A particularly large impact dispersion can result when the missile roll rate reverses direction, at which point the lift vector is momentarily stationary in space.[6-11] Impact error caused by roll-trim dispersion at constant roll rate varies approximately inversely with roll rate.[3] Therefore, roll-trim dispersion can be controlled by maintaining the roll rate at some reasonably steady nonzero value.

A statically stable missile is untrimmed when its angle of attack is large relative to its trim angle of attack due to body-fixed asymmetries. This condition can result from launch errors and is a common condition of a re-entry vehicle that has its angular momentum vector misaligned with its velocity vector at re-entry.[12] The ensuing motion is a transient, untrimmed condition that exists while the initial angle of attack converges to its steady trim value. In the untrimmed condition, the lift vector consists of two rotating components in the familiar epicyclic motion.[2] In the absence of aerodynamic disturbances, transverse center-of-gravity motion of a symmetric missile in epicyclic motion is confined to small amplitudes in spite of large oscillations in angle of attack and lift vector rotation rate. This is rather fortuitous, and if it were not the case, spinning missiles would be considerably less accurate than they are.

A potential source of impact dispersion is identified for certain forms of aerodynamic disturbances that cause lift nonaveraging with initially untrimmed, epicyclic motion. The disturbance moments are wind-fixed, in contrast to body-fixed trim moment variations that have been analyzed in conjunction with roll-trim dispersion. Wind-fixed moments can be classified as in-plane (pitch) or out-of-plane (yaw) in the classical Euler angle or plane-fixed formulation of missile motions.[12-16] Such moment perturbations have been observed to occur during boundary-layer transition of re-entry vehicles.[17-19] Out-of-plane moments also can occur from ablation time lag effects.[20] In this paper we show that both in-plane and out-of-plane moment perturbations can cause lift-nonaveraging dispersion. Unlike roll-trim dispersion, during which the lift vector precesses at the roll rate and generally is uncoupled from the trim variations, there is a strong coupling between lift precession rate and angle of attack during the motion response to wind-fixed moment perturbations. The roll rate in this case has little influence on dispersion. The coupling between precession rate and angle of attack occurs from gyroscopic effects when the motion is untrimmed and therefore epicyclic. A strong influence of initial motion conditions on the subsequent lift nonaveraging is shown, and worst-case disturbance moments are identified that can result in precession stoppage at angle of attack, which gives the greatest impact error.

Coordinate System and Dispersion Model

The moment equations of motion are analyzed in terms of the classical Euler angle coordinates shown in Fig. 1. It is assumed that the rate of trajectory bending is small compared with the vehicle angular rates, and that the contribution of angle of attack from lateral translation of the center of mass is small compared with the pitch angle θ. Omission of lateral translations generally is not valid with regard to pitch or yaw normal force damping, and a suitable approximation for this effect is included in the moment equations of motion.

The moment equations can be written approximately[12-14]†

$$\ddot{\theta}+2\zeta\omega\dot{\theta}+(\omega^2+\mu p\dot{\psi}-\dot{\psi}^2)\theta=M_p/I+(M_t/I)\cos\phi \qquad (1)$$

$$\theta\ddot{\psi}+2\zeta\omega\dot{\psi}\theta+(2\dot{\psi}-\mu p)\dot{\theta}=M_y/I+(M_t/I)\sin\phi \qquad (2)$$

$$\dot{p}=\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\phi}+\dot{\psi})=M_r/I_x \qquad (3)$$

Impact dispersion due to lift and precession variations can be obtained simply by considering the transverse velocity produced from lift nonaveraging in a plane perpendicular to the nominal flight path. The transverse velocity $V=v+iw$ obtained by considering the vehicle as a point mass in this plane under the action of a rotating lift force is defined by (Fig. 2):

$$V=V(0)-\frac{iL_\theta}{m}\int_0^t \theta e^{i\psi}\mathrm{d}t \qquad (4)$$

where the factor $-i$ is included for consistency of the angular and translational coordinates, and the integral is taken over a time increment that includes the perturbations in θ and ψ that cause lift nonaveraging. The lift force derivative $L_\theta=C_{L_\theta}qS$ is, in general, slowly varying and assumed constant over the time duration of the θ and ψ disturbances. The cross-range dispersion that results from a net transverse velocity increment ΔV is approximately

$$r=s|\Delta V|/u \qquad (5)$$

where s is the target range from the point of disturbance and ΔV is the net change in the mean transverse velocity produced by the moment disturbance. The aerodynamic jump J or trajectory deflection due to lift nonaveraging is defined by[3]

$$J\equiv\lim_{t\to\infty}|\Delta V(t)|/u \qquad (6)$$

†In the wind-referenced coordinate system, pitch and yaw moments are "in-plane" and "out-of-plane" moments, respectively, with respect to the precessing plane of total angle of attack.

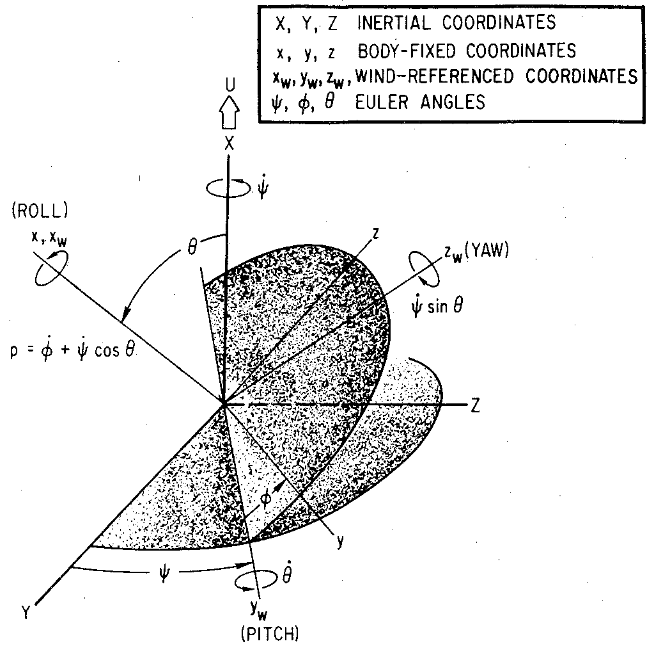

Fig. 1 Euler angle coordinates.

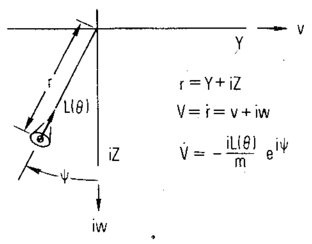

Fig. 2 Transverse velocity.

The transverse velocity can be expressed in terms of the complex angle of attack defined by

$$\xi=\beta+i\alpha=i\theta e^{i\psi} \qquad (7)$$

which, from Eq. (4), gives

$$V=V(0)-\frac{L_\theta}{m}\int_0^t \xi\mathrm{d}t \qquad (8)$$

We can write the equations of motion in terms of ξ by substituting $\theta=-i\xi e^{-i\psi}$ in Eqs. (1) and (2), which yields

$$\ddot{\xi}+(2\zeta\omega-i\mu p)\dot{\xi}+\omega^2\xi=m_w e^{i\psi}+im_t e^{i\Phi} \qquad (9)$$

where m_w is a complex wind-fixed moment, and m_t is a body-fixed trim moment. The roll angle Φ is defined for small θ by the general expression

$$\Phi=\phi+\psi=\int p\mathrm{d}t+\text{const} \qquad (10)$$

We consider various forms of pitch and yaw moment disturbances M_p, M_y with constant roll rate ($M_r=0$) and a symmetric missile ($M_t=0$). The problem reduces to solving the coupled pitch and yaw moment Eqs. (1) and (2) for θ and ψ or Eq. (9) for ξ and integrating the results in Eq. (4) or (8) to obtain the net transverse velocity increment ΔV. Before proceeding with this analysis, it is of interest, for comparison, to derive the aerodynamic jump or trajectory deflection of a rolling, trimmed missile subjected to impulsive body-fixed moment disturbances.

Roll-Trim Dispersion

Consider the response of a rolling trimmed missile to a step increase in body-fixed moment from M_t to $M_t+\Delta M_t$. We can solve Eq. (9) for the response of ξ to a suddenly applied moment $M_t+\Delta M_t$ with arbitrary initial conditions $\xi(0)$ and $\dot{\xi}(0)$, integrate this result in Eq. (8) for the resulting transverse velocity, and subtract from this the transverse velocity due to a step M_t with the same initial conditions in order to obtain the net velocity increment due to ΔM_t. It is apparent from the linearity of Eq. (9) that the homogeneous solution for $\xi(t)$, which depends only on the initial conditions, will drop out, and the net transverse velocity increment will depend only on the particular solution $\Delta\xi(t)$ with the moment increment ΔM_t according to

$$\Delta V=-\frac{L_\theta}{m}\int_o^t \Delta\xi(t)\,\mathrm{d}t \tag{11}$$

The particular solution to Eq. (9) for $\Delta\xi(t)$ with $M_w=0$ and in absence of damping is found to be

$$\Delta\xi(t)=\frac{i\Delta m_t}{(\omega^2-p^2)}\left[\frac{\sqrt{\omega^2-p^2}}{\omega}\sin(\omega t-\gamma)+e^{ipt}\right] \tag{12}$$

where $\Delta m_t \equiv \Delta M_t/I$ and $\gamma\equiv\tan^{-1}(i\omega/p)$. The transverse velocity increment obtained by integrating Eq. (12) in Eq. (11) is

$$\Delta V=\frac{L_\theta \Delta m_t}{mp\omega^2}\left[1+\frac{ip}{\sqrt{\omega^2-p^2}}\cos(\omega t-\gamma)-\frac{\omega^2}{\omega^2-p^2}e^{ipt}\right] \tag{13}$$

Since the sinusoidal terms produce bounded oscillations for nonzero p and ω, the aerodynamic jump or trajectory deflection results from the constant term only, which, from Eq. (6), gives

$$J_{\text{step}}=\frac{L_\theta \Delta m_t}{mpu\omega^2} \tag{14}$$

If we define $\Delta\theta_t \equiv \Delta m_t/\omega^2(1-p^2/p_{cr}^2)$ as the rolling trim angle of attack due to the moment increment Δm_t, then the jump due to a trim step can be written

$$J_{\text{step}}=\frac{\Delta L}{mpu}(1-p^2/p_{cr}^2) \tag{15}$$

where $\Delta L = L_\theta\,\Delta\theta_t$ is the lift step due to $\Delta\theta_t$. This deflection occurs in a plane that leads by 90° the direction of the lift increment with respect to the rotating lift vector; i.e., the lift increment ΔL is in the $-iZ$ direction, whereas the deflection is in the $+Y$ direction. The foregoing analysis demonstrates that the epicyclic component of motion produced from an impulsive change in body-fixed moment at constant roll rate does not contribute to trajectory deflection, and roll-trim dispersion can be obtained simply from the integral

$$\Delta V=-\frac{iL_\theta}{m}(1-p^2/p_{cr}^2)\int_o^t \Delta\theta_t(t)e^{ipt}\mathrm{d}t \tag{16}$$

where $\Delta\theta_t(t)$ is the quasisteady perturbation in rolling trim angle of attack produced by a perturbation in body-fixed moment. If, rather than a step change in θ_t, the change occurs in the form of a ramp from θ_t to $\theta_t+\Delta\theta_t$ in Δt sec, the trajectory deflection is found to be

$$\lim_{t\to\infty}\Delta V(t)/u=\frac{i\Delta L}{mp^2u\Delta t}(1-p^2/p_{cr}^2)(1-e^{ip\Delta t}) \tag{17}$$

which has a magnitude

$$J_{\text{ramp}}=\frac{\Delta L}{mpu}(1-p^2/p_{cr}^2)\frac{|\sin(p\Delta t/2)|}{(p\Delta t/2)} \tag{18}$$

Since $\Delta L(1-p^2/p_{cr}^2)/mpu$ is the jump due to a step ΔL, the ratio of the jump due to a ramp to that due to a step is the function

$$\frac{J_{\text{ramp}}}{J_{\text{step}}}=\frac{|\sin(p\Delta t/2)|}{(p\Delta t/2)} \tag{19}$$

which is plotted in Fig. 3 as a function of the ramp duration Δt. The envelope of this function varies inversely with Δt and demonstrates that an "impulsive" change in lift must occur in about or less than the time duration of the lift vector rotation period in order to have a large effect on dispersion. We now examine forms of the wind-fixed moment m_w that can produce large trajectory deflections during untrimmed motion. It is convenient for this analysis to use the Euler angle equations of motion, Eqs. (1-3).

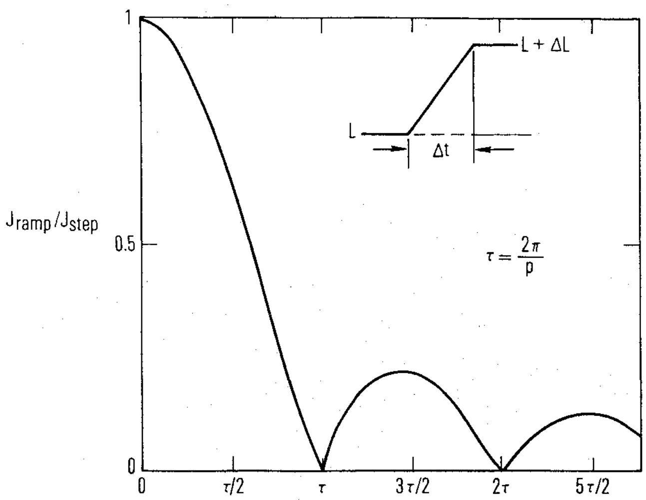

Fig. 3 Ratio of jump due to ramp change in lift to jump due to a step as a function of ramp duration.

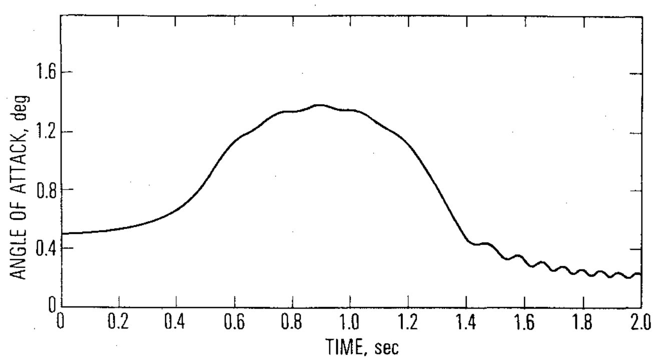

Fig. 4 Angle-of-attack buildup.

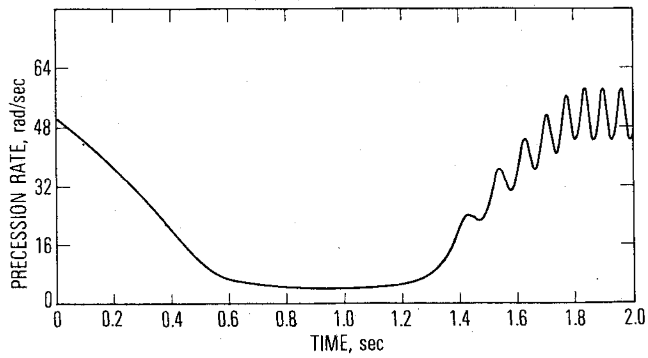

Fig. 5 Precession rate decrease.

In-Plane Moment Disturbances

In the absence of a trim moment M_t and an out-of-plane moment M_y, the yaw equation, Eq. (2), can be written

$$\frac{d}{dt}(\dot{\psi}\theta^2)+2\zeta\omega(\dot{\psi}\theta^2)-\frac{\mu p}{2}\frac{d}{dt}(\theta^2)=0 \quad (20)$$

which, for constant roll rate and excluding damping, can be integrated to yield

$$\left(\dot{\psi}-\frac{\mu p}{2}\right)\theta^2=\text{const} \quad (21)$$

For angles of attack somewhat greater than those dictated by body-fixed trim asymmetries (assumed zero here), the quasisteady value of the precession rate parameter $\dot{\psi}-\mu p/2$ is approximately equal to the pitch frequency ω for a slowly rolling missile.[12] Under these conditions θ and $\dot{\psi}$ are coupled according to

$$\left(\dot{\psi}-\frac{\mu p}{2}\right)\theta^2\approx\omega\theta_o^2 \quad (22)$$

where θ_o is the quasisteady value of the angle of attack just prior to a moment perturbation. Even with aerodynamic damping, the exponential decay in $\dot{\psi}\theta^2$, described by the first two terms in Eq. (20), is sufficiently slow that Eq. (22) is a reasonable approximation for the time duration of the disturbances in θ and $\dot{\psi}$ that are analyzed. For in-plane moments only, the problem reduces to solving Eq. (1) for some prescribed pitch moment $M_p(\theta,t)$, subject to the constraint, Eq. (22), and integrating the results for θ and $\dot{\psi}$ in Eq. (4) to obtain the transverse velocity increment.

Equations (15) and (19) indicate that, for maximum dispersion due to an incremental change $\Delta\theta$ in angle of attack at constant precession rate $\dot{\psi}$, the change in θ must occur in a time increment Δt that is about or less than the precession period $2\pi/\dot{\psi}$. The equations also indicate that the magnitude of the dispersion for a given change $\Delta\theta$ is inversely proportional to the precession rate $\dot{\psi}$. Unlike roll-trim dispersion, where arbitrary changes in θ occur at constant precession rate $\dot{\psi}=p$ or vice versa with θ and p uncoupled, the untrimmed dynamics characterized by Eq. (21) show a strong coupling between θ and $\dot{\psi}$ that is essentially independent of the roll rate. It is apparent from Eq. (21) that any pitch moment disturbance that causes a momentary angle of attack appreciably larger than the initial quasisteady angle of attack θ_o can reduce the precession rate to a value so low that the lift vector becomes stationary, for practical purposes, in a time increment that is short compared with the precession period. We define $\Delta\theta_p$ as the angle of attack increment caused from some pitch moment disturbance $M_p(\theta,t)$. The quasisteady precession rate, from Eq. (22), is then

$$\dot{\psi}_p=\omega\left(\frac{\theta_o}{\theta_o+\Delta\theta_p}\right)^2 \quad (23)$$

and the precession period τ is

$$\tau=2\pi/\dot{\psi}_p \quad (24)$$

For example, let $\theta_o=0.33$ deg and $\Delta\theta_p=1$ deg with the pitch frequency $\omega=40$ rad/sec. The quasisteady precession rate decreases to 0.4 rps with a period of 2.5 sec. If we assume that the trim exists for a time increment Δt that is small compared with τ, then we can ignore the exponential term in Eq. (4), and the aerodynamic jump is approximately

$$J=\frac{L_\theta(\theta_o+\Delta\theta_p)\Delta t}{mu} \quad (25)$$

This expression is independent of the roll rate and indicates, from a comparison with Eq. (15), that trajectory deflections of the same order of magnitude or larger than those occurring from impulsive body-fixed moments can occur from in-plane moment disturbances, depending on the initial conditions. The ratio of the jump due to an in-plane moment to that due to a body-fixed trim step is $(\theta_o+\Delta\theta_p)p\Delta t/\Delta\theta$, where p is the roll rate of the trimmed vehicle, and the in-plane moment duration Δt is small compared with the precession period τ defined by Eq. (24). For the foregoing example in which $\tau=2.5$ sec, a 1-deg in-plane moment disturbance of 0.25-sec duration will produce approximately four times the trajectory deflection caused by an equivalent body-fixed trim angle-of-attack step with a trimmed missile rolling at 2 rps. The in-plane moment disturbance need not be impulsive to produce large dispersion. For a sufficiently low precession rate $\dot{\psi}_p$ during the moment disturbance, from Eq. (23), depending on θ_o and the in-plane moment increment $\Delta\theta_p$, the trajectory deflection can become prohibitively large if the moment persists for a sufficient time duration. This is illustrated in Figs. 4-6, which show results of a numerical integration of the equations of motion for a gradual buildup and decay of in-plane moment of the form

$$M_p(\theta,t)=M_{p_{max}}\sin\frac{\pi\theta}{\Theta}\sin\frac{\pi t}{\tau_p},\quad 0\le\theta\le\Theta,\ 0\le t\le\tau_p$$
$$=0,\qquad \theta>\Theta,\ t>\tau_p \quad (26)$$

Table 1 Missile and motion parameters

$m=8$ slugs	$\zeta=0.011$
$I=14.7$ slug-ft^2	$M_{p_{max}}/I=60$ sec^{-2}
$\mu=0.0959$	$\Theta=3$ deg
$S=2.18$ ft^2	$\tau_p=1.8$ sec
$L_\theta=2615$ lb/deg	$p=2$ rps
$\omega=50$ rad/sec	

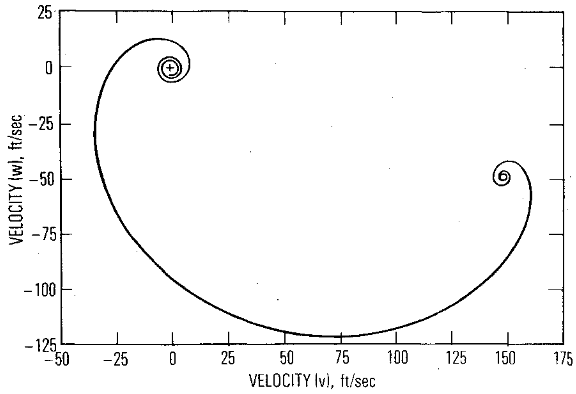

Fig. 6 Transverse velocity profile.

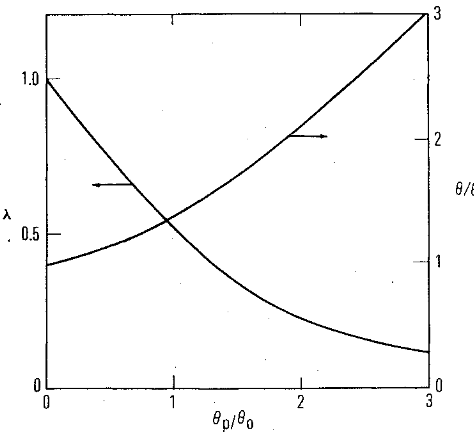

Fig. 7 Influence of initial angle of attack and trim angle on precession rate.

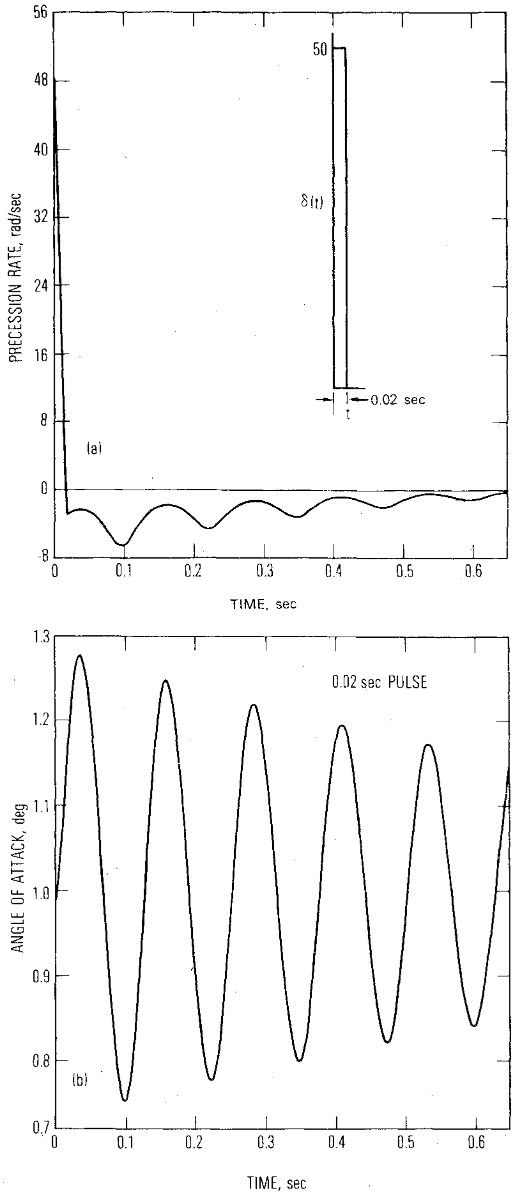

Fig. 8 Precession rate response (a) and angle-of-attack response (b) to pitch and yaw moments for 0.02-sec pulse duration.

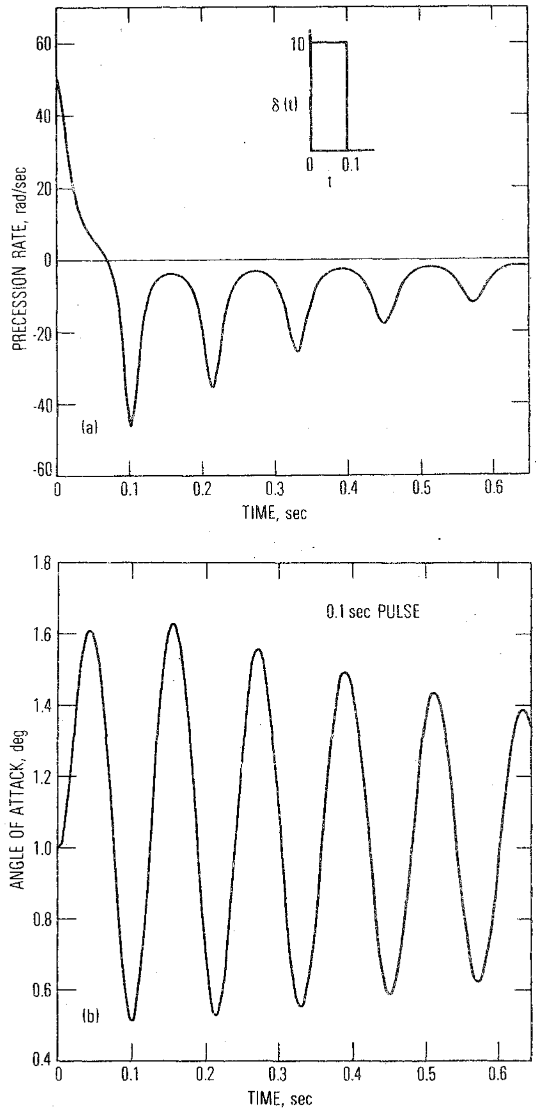

Fig. 9 Precession rate response (a) and angle-of-attack response (b) to pitch and yaw moments for 0.1-sec pulse duration.

where the half-period τ_p of the moment disturbance is considerably greater than the pitch period $2\pi/\omega$ of the missile. Parameters used in the numerical example are shown in Table 1. The values selected for $M_{p\max}$ and Θ result in a static moment with a trim angle at $\theta \approx 1.4$ deg. This form of static moment is similar to that frequently described as an S-curve or type (c) moment,[21] and effects of this type of moment on the dispersion of submunitions in steady trimmed flight have been investigated.[8-11] The angle-of-attack buildup (Fig. 4) starts from an initial angle of attack of 0.5 deg; the precession rate decrease is shown in Fig. 5. The initial precession rate is approximately equal to the pitch frequency ω, and the peak angle of attack of 1.4 deg corresponds to a trim increment $\Delta\theta_p$ of 0.9, deg, which, from Eq. (23), gives a predicted minimum precession rate $\dot{\psi}_p$ of approximately 6 rad/sec. The actual minimum $\dot{\psi}$ in Fig. 5 is slightly lower because of aerodynamic damping. The net transverse velocity increment (Fig. 6) is approximately 156 fps. By comparison, a 0.9-deg body-fixed trim step $\Delta\theta$ applied to the same missile rolling at 2 rps would produce a transverse velocity increment, from Eq. (13), of only 25 fps. Unlike roll-trim dispersion, the deflection of an initially untrimmed missile because of in-plane moment disturbance is strongly dependent on the initial angle of attack and independent roll rate. Hence, roll control would have little effect in reducing such dispersion.

We can generalize the preceding result for the response of an initially untrimmed missile to a static moment that has a trim angle θ_p. The in-plane moment increment required is

$$M_p = \omega^2 \theta_p I \tag{27}$$

In the absence of other moments, and neglecting small roll-rate terms proportional to μp, we can write the quasisteady values of Eq. (1) and (2) for $\dot{\theta} = \ddot{\theta} = 0$

$$(1-\lambda^2)\theta = \theta_p \tag{28}$$

$$\lambda \theta^2 = \theta_o^2 \tag{29}$$

The precession ratio obtained from Eqs. (28) and (29) is defined by

$$\lambda^2 = 1 - \lambda^{1/2}(\theta_p/\theta_o) \tag{30}$$

as shown in Fig. 7 together with θ/θ_o as a function of θ_p/θ_o. For θ_p of the order of or less than θ_o, the precession reduction is not significant with respect to lift nonaveraging. However, a small additional impulsive out-of-plane moment can drive the precession rate to zero for arbitrary initial conditions.

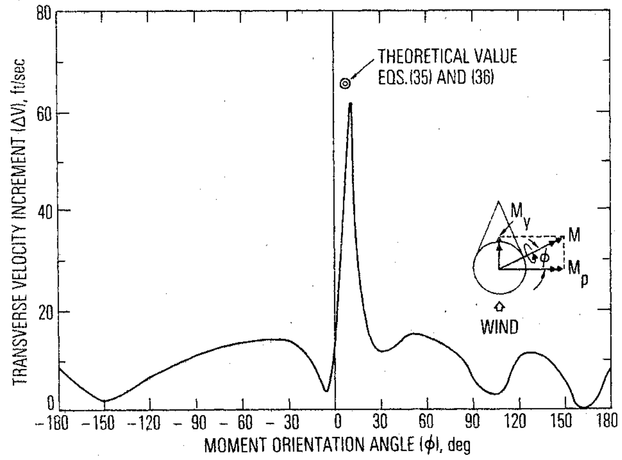

Fig. 10 Transverse velocity response to 0.2-sec rectangular moment pulse vs moment orientation angle.

Wind-Fixed Moment Conditions for Precession Stoppage

The most adverse condition for dispersion of a spinning missile occurs when the lift vector becomes stationary while the missile has a finite angle of attack. This condition occurs momentarily when a slightly asymmetric missile in rolling trimmed flight has a roll-rate reversal as a result of a negative roll torque. The magnitude of the trajectory deflection or aerodynamic jump depends on the magnitude of the trim angle of attack and the rate at which the missile passes through zero roll rate.[6-9] An even worse condition can occur for a symmetric missile at constant roll rate that is initially untrimmed and subjected to impulsive moments. We derive moment conditions that can result in precession stoppage while the missile has a nonzero angle of attack and steady roll rate.

If we assume the missile is at a quasisteady angle of attack $\theta_p \approx$ const caused by an in-plane moment M_p, and apply an out-of-plane (yaw) moment at $t=0$, we can write the yaw equation, Eq. (2)

$$\frac{d\dot{\psi}}{dt} + 2\zeta\omega\dot{\psi} = M_y(t)/I\theta_p \tag{31}$$

The Laplace transform of Eq. (31) is

$$(s+2\zeta\omega)\dot{\psi}(s) - \dot{\psi}(0) = M_y(s)/I\theta_p \tag{32}$$

which indicates that the precession rate will decrease to and remain zero, i.e., $\dot{\psi}(t) = 0$, $t>0$, for a yaw moment:

$$M_y(t) = -I\theta_p\dot{\psi}(0)\delta(t) \tag{33}$$

This result has physical significance in that the yaw moment described by Eq. (33) is that impulse that will bring the initial precession motion to rest. We can approximate $\delta(t)$ by a rectangular pulse of duration t_p and magnitude $1/t_p$ such that

$$\int \delta(t)\,dt = 1$$

If we assume that the missile is precessing initially at $\dot{\psi}(0) = \omega$, then the yaw moment required to stop the precession is approximately

$$M_y(t) = -I\theta_p\omega/t_p, \quad 0 \le t \le t_p \tag{34}$$

The pitch moment required to sustain the quasisteady angle of attack at θ_p for $t>0$ with $\dot{\psi}=0$ is given by Eq. (27). Computer simulation results of precession stoppage at angle of attack are shown in Figs. 8 and 9 for yaw moment pulse durations of 0.02 and 0.1 sec, respectively, initial precession rate of 50 rad/sec, and $\theta_p = 1$ deg. If we assume that both pitch and yaw moments are components of a wind-fixed moment pulse of duration t_p oriented at some angle ϕ from the windward meridian (such that $\phi=0$ is an in-plane moment), then the ratio of the out-of-plane to in-plane moment required to stop the precession for t_p sec at an angle of attack θ_p, from Eqs. (27) and (34), is approximately

$$M_y/M_p = -1/\omega t_p \tag{35}$$

This should cause a transverse velocity increment

$$\Delta V = L_\theta\theta_p t_p/m \tag{36}$$

based on complete precession stoppage over the pulse duration t_p. For example, for $\omega = 50$ rad/sec and $t_p = 0.2$ sec, the yaw moment required to stop the precession is 10% of the pitch moment, which corresponds to a moment pulse rotated 5.7 deg out of plane of the lift vector. For $\theta_p = 1$ deg with the missile parameters of Table 1, the velocity, increment from Eq. (36), is found to be 65.4 fps. The velocity increment obtained from a numerical solution of the missile response to

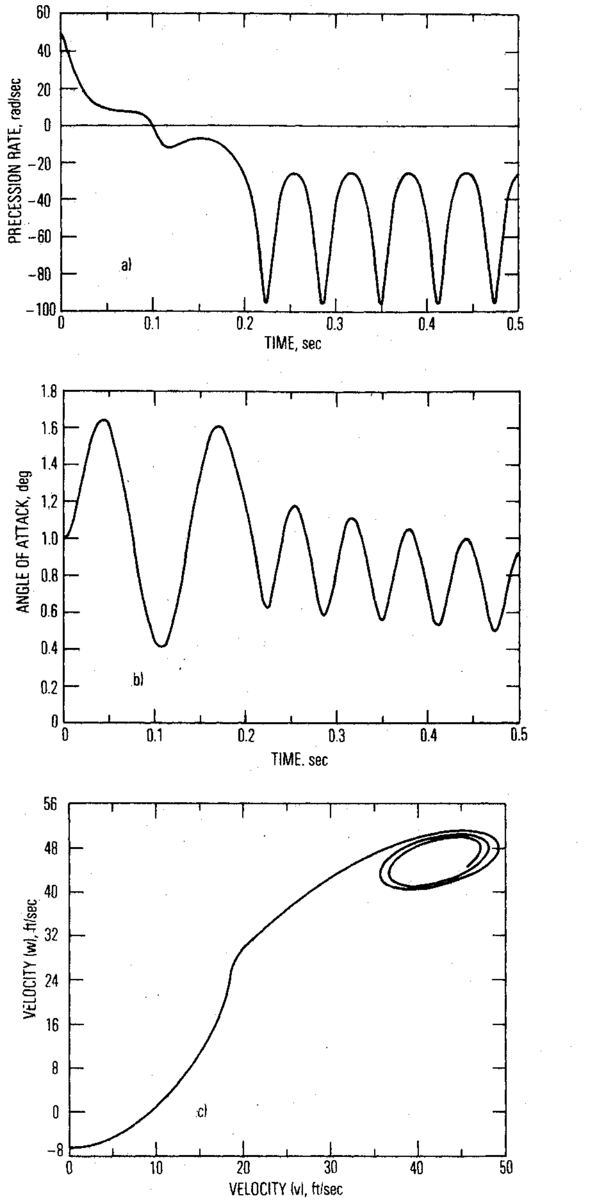

Fig. 11 Precession rate (a), angle-of-attack (b), and transverse velocity (c) profiles for 0.2-sec moment pulse oriented at $\phi = 8.9$ deg.

a 0.2-sec duration moment pulse is shown in Fig. 10 as a function of the moment orientation angle ϕ. Also shown in Fig. 10 is the theoretical value calculated from Eqs. (35) and (36). The maximum dispersion is found to be approximately 95% of the theoretical value based on complete precession stoppage. It occurs from a moment pulse oriented 8.9 deg out of the wind plane compared with 5.7 deg predicted from Eq. (35). This discrepancy in orientation angle results from approximating the unit impulse function in Eq. (33) by $1/t_p$ to obtain Eq. (34) and is apparent from the influence of pulse duration on the precession rate in Figs. 8 and 9. The precession rate, angle of attack, and transverse velocity histories for the 8.9 deg moment orientation that gives maximum dispersion are shown in Fig. 11. It is somewhat surprising that the quasisteady theory with a 0.2-sec rectangular pulse approximation to the unit impulse function predicts the maximum dispersion so closely in view of the actual nonzero precession rate during much of the pulse duration (Fig. 11a).

Summary and Conclusions

It has been demonstrated that moment perturbations of an initially untrimmed missile can produce cross-range dispersion due to lift nonaveraging that is essentially independent of roll rate. The lift vector precession rate in untrimmed, epicyclic motion has modal values approximately equal to the missile pitch frequency for a slowly spinning, statically stable missile. Certain combinations of in-plane and out-of-plane moments can adversely affect the precession rate and even cause precession stoppage in which the lift vector is momentarily stationary in space at nonzero roll rate. Dispersion due to such moment combinations can be substantially greater than that caused by comparable body-fixed moment perturbations of a rolling, trimmed missile.

A quasisteady analysis of lift vector precession rate and angle-of-attack coupling indicates a strong dependence of dispersion on the initial motion conditions and reveals the form of moment perturbations required for worst-case precession behavior and dispersion. An impulsive moment oriented to give a small component out of plane of the wind is very effective in causing precession stoppage. Precession stoppage also can occur for a slowly varying in-plane moment that results in a static trim angle, depending on the magnitude of the initially untrimmed angle of attack. In either case, very large trajectory deflections can occur. Results of the theory have been verified by numerical integrations of the equations of motion. Roll control is ineffective for limiting this newly identified source of dispersion, and effective control must provide some means of maintaining a steady, nonzero precession rate.

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